

Laws and Models in Physics vs Biology and the Continuum vs Discrete Interplay

Fribourg University

Giuseppe Longo

Centre Cavallès, République des Savoirs, CNRS,
Collège de France et Ecole Normale Supérieure, Paris, and
Dept. of Integrative Physiology and Pathobiology, Tufts Univ., Boston

www.di.ens.fr/users/longo

Biology and its "close" sciences

- 1) **Physical** Theories and their Mathematics
- 2) **Human** Sciences

Biology: borrow the **method** from 1:

observation of nature and experiences (prepared)

Yet, a **historical science**, as 2.

Is there a common notion of “law” ?

Laws of Nature?

No explicit **wording** of *Laws* in Galileo and Descartes,
in reference to nature.

Similarly in Kopernic and Kepler (Roux, 2009).

According to Needham (1951) :

The scientific revolution will explicitly refer to the notion of law
(of nature) only after the **unified Monarchies** (Modern
European States) will adopt the "*law of the (absolute) king*".

Thesis :

"**Laws of nature**" will be explicitly defined as such only after
their explicit mathematical writing (equations) in *suitable
spaces*

The invention of Space

Italian Geometric perspective

It originated in the representation of the Presence of God in
annonciations

Infinity in Annonciations in Italian paintings (Arasse vs Panofski):

Religious Infinity organised the *human space* of Renaissance

Sara Longo, “La “perspective” de l’Annonciation”, **Studiolo X**, 2010 (in 2013).

G. Longo. “*Mathematical Infinity "in prospettiva" and the Spaces of Possibilities*”. In
“**Visible**”, a Semiotics Journal, n. 9, 2011, downloadable.

Infinity in Mathematics, via Paintings

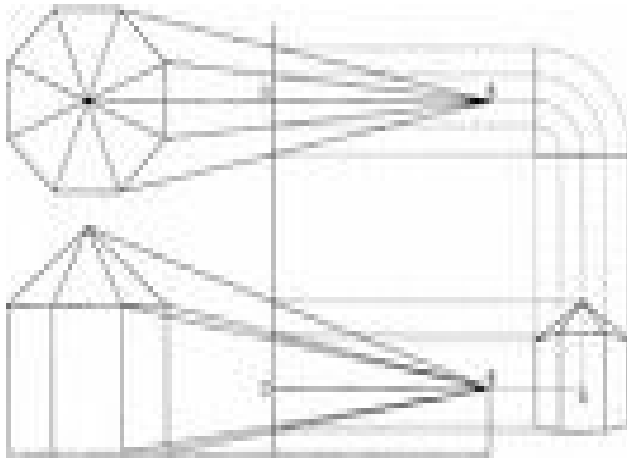
Piero della Francesca: “Annunciation”, God vs. Mary, 1470



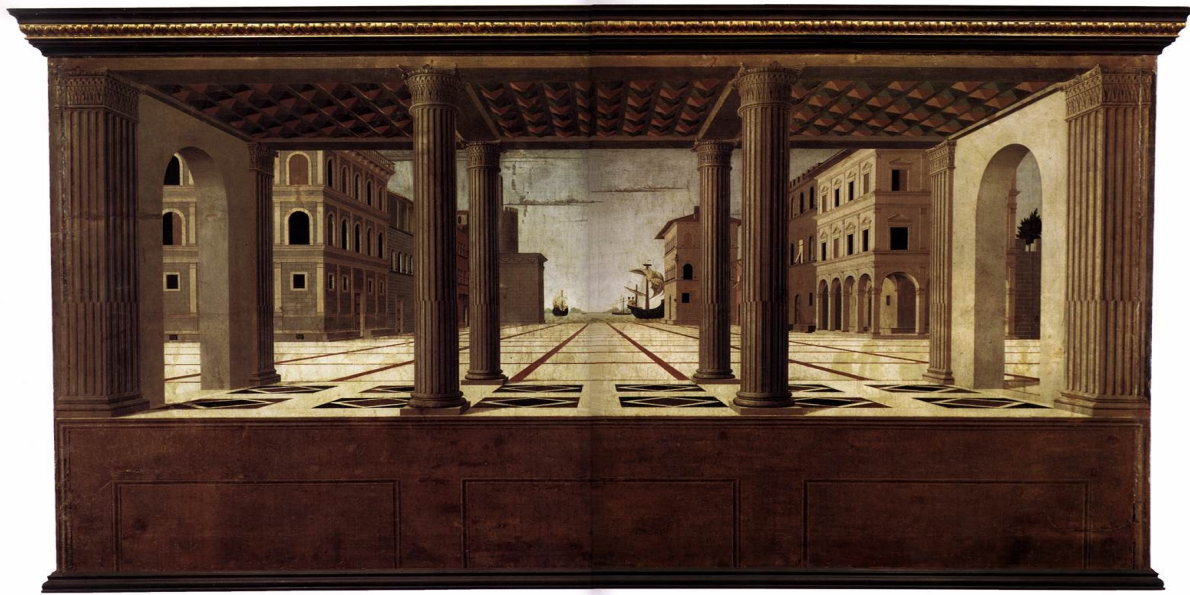
Infinity and Space in Mathematics, via Paintings

From infinity “in the painting” to the mathematics of space:
Piero della Francesca, De perspectiva pingendi (~1450)

Construction of a “point of view”,
(Practical) **projective Geometry**:



Space is pre-given, it may be empty



Francesco di Giorgio Martini, 1490

« Space exists before the bodies placed in it, thus it must be graphically given first ».

Pomponius Gauricus, *De sculptura*, Florence, 1504.

Later: **Descartes** (1596-1650) *and*
Desargues (1591-1661)

PHYSICAL SPACES

Aristotle, ... Galileo only analyze *trajectories*.

Modern Mathematical Spaces in Physics, a later invention:

1 – Descartes spaces:

The space of all possible trajectories, *defined by symmetries*

2 - Galileo's inertia:

A momentum conservation law

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1 + 2 : Galileo's **relativity**: the *symmetry group* ($x' = x - vt$),

transformations that preserve the physical invariants

3 - Einstein's **Relativity**: *Lorentz-Poincaré's symmetry group*

($\Delta x' = (\Delta x - v\Delta t)/\sqrt{1 - v^2/c^2}$) and ($\Delta t' = (\Delta t - v\Delta x/c^2)/\sqrt{1 - v^2/c^2}$)

Physical Determination and the Laws

Newton's equations are

given in pre-defined (empty) spaces

Kant: space (and time):

- A priori intuitions
- Conditions of possibility for physical knowledge

One needs the parameters (space and time) in order to write equations:

**EQUATIONS ARE THE RIGOUROUS EXPRESSION OF THE
LAWS OF NATURE**

<http://www.di.ens.fr/users/longo/CIM/ProjetLongo2014-17.pdf>

XVIII century: Lagrange, Laplace

PHASE SPACES IN PHYSICS, XIX century

The construction of **phase space** in Physics (*step 2*):

add the *invariants* that matter:

all pertinent **observables** and **parameters**.

- **Boltzmann, Poincaré** (1880-90): analyze the trajectory in
- “position, momentum” (**p,q**) spaces (till now: canonical “*phase space*” of physics – alternatively: **E, t**)

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- **Thermodynamics**:
 - the p, V, T space of the thermodynamic trajectory
 - a new observable: **entropy**.

PHASE SPACES AND SYMMETRIES IN PHYSICS

Thus the *phase space* is proposed on the grounds of

- **Symmetries** (beginning with Descartes axes)
- **Invariants of the trajectories** (momentum), thus, again, *symmetries* (Noether's theorems)

Similarly for E, t (as for p, q ; all “conjugated variables” in QM)

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“*Put in the “background”* the intended space of trajectories - the pertinent one for the trajectories:

- **parameters** (space, time) *and*
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Also in **QM**: the “*wave or state function*” goes along a trajectory in **Hilbert Spaces** (Schroedinger's choice of the phase space)

PHASE SPACES, LAWS AND PREDICTION IN PHYSICS

Classical and Quantum systems are "**State Determined**", i.e.

The *state* and *the equations* fully determine the trajectory (of a *law of probability* in QM)

There may be **unpredictability**:

- Deterministic unpredictability (**classical**)
- Intrinsic unpredictability (**quantum**: Schrödinger)
- **Statistical** physics: unpredictable number of particles (dimensions), yet observable properties do not change

PHASE SPACES, LAWS AND PREDICTION IN PHYSICS

In physics, the mathematical *present* determination allows to frame the future developments, by making **predictions** (*solving equations, computing evolution functions*) and/or by **assigning probabilities** to the different future outcomes, in presence both of classical and of quantum-mechanical unpredictability.

THIS IS THE MATHEMATICAL FRAME FOR THE
LAWS (OF NATURE)

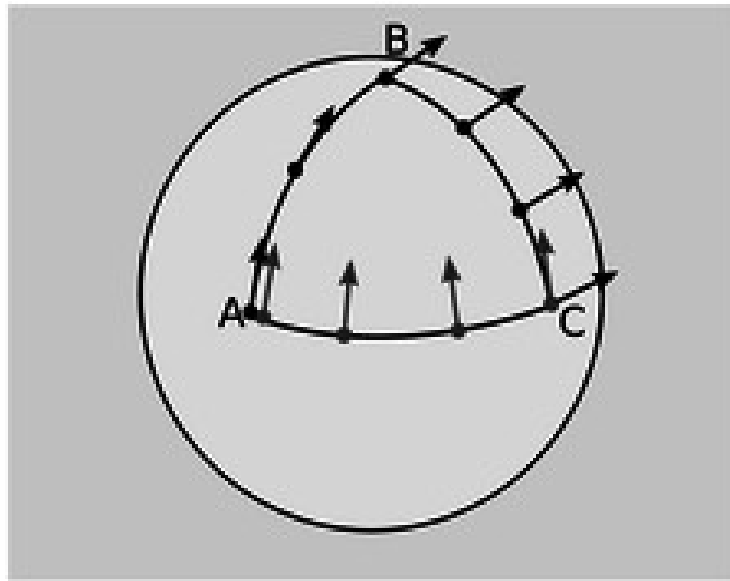
In fully per-determined (pre-given) spaces

State Determined Systems: no history

(no relevance of the past)

PATH DEPENDENCE IN PHYSICS

Levi-Civita in Riemann's geometry: the orientation of the vector (e.g. momentum) from A to C **depends on the path** followed:



However, add momentum and obtain the (momentum, position) phase space, then back to a SDS

PATH DEPENDENCE IN PHYSICS

More **history dependence** in physics:

- Thom's Theory of Catastrophes (Arnold, 1992),
- Critical phase transitions in Theory of Criticality (Binney et al., 1992)

The **nature and order** of the *singularities* and *transitions* may matter in the determination of the final result.

Or, the presence of a singularity or of a critical point may “deform” a geodesic and lead to a different state.

However, in general, different paths, when viewed in the *pertinent phase space* as above, do not modify the state determination nor the future behavior of the system.

That is, **in a *suitable phase space*, one gets SDS, as usual.**

(The discussion here and below is to be found in

"How Future Depends on Past Histories in Systems of Life", in prepar., downloadable from: <http://www.di.ens.fr/users/longo>)

PROCESSES VS. HISTORY IN PHYSICS

A Planetary System, a hurricane, a flame, a Benard chemical cycle ... are **processes** with a time (they change)

But ... they go always along the **same process**, they are **identical** since 4 billion years.

Processes may have *an internal history*, they are ***not historical***

These *far-from-equilibrium systems* are (self-) **organization** of flows of energy and/or matter.

An *organism* **uses** and constraints by its own structure, the flows of energy and matter.

MEMORY AND HISTORY IN COGNITION

In animal cognition:

(pre-conscious) **retention** (and conscious *memory*) are forms of construction of invariants for the purposes of action.

The brain *singles out* some features that it considers crucial for the ongoing activity; it *excludes* by this the details that are irrelevant.

Retention (and memory) is **selective** and *forgetful*:

it forms by this a relatively stable invariant as the result of manifold activities, by selecting and thus forgetting

Animal retention and memory exist **for the purposes of action**, beginning with the simplest animal movement [Berthoz, 1997]

MEMORY AND HISTORY IN COGNITION

In summary, the **network of past experiences produces a cognitive invariant**, as the result of a distilled sound activity in different frames.

This is used for future action: in this sense

–**The future depends on the past** (true path dependence)

The past is constantly **re-interpreted**, with a goal, the ongoing action.

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(Edelman, Tononi, 2000) considers the action of recalling as produced by the **brain which puts itself in an already lived state**.

This brain's state is the result of a **past activity in a possibly no longer existing context**, where it had a specific sense, e.g. the “meaning” of a former action.

MEMORY AND HISTORY IN SYSTEMS OF LIFE

Recall: *physical* (main) observables:

energy, momentum ... entropy ...

In *Biology*: Darwin's theory focuses on

phenotypes, organisms and species (*the proper observables*).

Darwin's two fundamental principles:

- *descent with modification*
- *selection*

apply to inheritance (in/of observables).

Evolution: a historical science.

MEMORY AND HISTORY IN SYSTEMS OF LIFE

Example of inherited invariant:

DNA functions may be **understood only in a relational way**:

it works and we can make biological sense of it, only in relation to a context, a cell living in a viable ecosystem.

DNA expression depends on

1. (de-)methylation
2. changing physical constraints:
 - on the chromatine,
 - on the nucleus membrane
3. the global epigenetic context (the cell, the tissue ...)

(Bécavin et al, 2012), (Desprat et al, 2008), (Fernandez-Sanchez et al, 2010) ...

MEMORY AND HISTORY IN SYSTEMS OF LIFE

DNA is thus the result of a history that "determined" it, in a relational way.

DNA today's structure *as well as* its current potentialities (possible expressions), **thus the future, depend on the future context and how the past relational structure that determined it.**

The **biological meaning** of any component of an organism (e.g. organ or function) depends on its **context** and its **past history** (e.g. the eye ... the bladder of a lung-fish ... Gould's **exaptation**).

Its **future re-use** can only be thought in reference to its previous constitution in a past context.

Laws in continua vs instructions in discrete data types

Continuous dynamics and the Computer

Continuity of *mathematical models* crucially steps in:

- **approximation** (an open interval of *measurement* or of the initial/border conditions)
- various forms of *instability, criticality, symmetry breakings* ...

Key issue: Discret (space-time) dynamics *are not* an approximation of *non-linear* continuous dynamics.

“It might be possible, however, to treat a *few particular cases* in detail with the aid of a digital computer.

The essential disadvantage of the method is that one only gets results for particular cases” (Turing, *Morphogenesis*, 1952, p. 72)

Sensitivity of the dynamics implies **divergent trajectories**, *yet* ...

Today's Shadowing Theorem: the “reverse” approximation

Computational problem: the **round-off**

Shadowing Theorem for hyperbolic dynamical systems (D, f, m)

For any x_0 and δ there is an ε such that, for any ε -approximated (or rounded-off $\leq \varepsilon$) trajectory, there is one in the continuum which goes δ -close to it, at each step.

Informally:

Given a “sufficiently regular” non-linear iterated function system, *any discrete (space-time) trajectory* can be actually approximated by a continuous one (**but, in general *not the converse!***)

Or ... *there are so many continuous trajectories*, that, given a discrete trajectory, you can find a continuous one which goes close to it, see:

Pilyugin, S.Yu. (1999). *Shadowing in Dynamical Systems*.

Lecture Notes in Math. 1706, Springer-Verlag, Berlin.

Digital computing and identical iteration

The **dice** or a **double pendulum** (dependence on minor variations of the initial conditions) know very well where they will go: along a **unique physical geodetics**, extremizing a Lagrangian action, according to Hamilton principle,
yet **highly unpredictable, never iterating.**

Instead, just push the **restart** button in a computer simulation and the most chaotic dynamics will iterate exactly, e.g., the double pendulum:

<http://www.mathstat.dal.ca/~selinger/lagrange/doublependulum.html>

One may « **imitate** » randomness (pseudo-random, or **borrowed** from outside ...): an imitation with **no understanding.**

Solving equations

Invention of **Galois**, **Lie** groups ...

To solve equations: **transfer symmetries** in equations to further symmetries, or prove **symmetry breaking** (non-analyticity, Poincaré)

In **Category Theory**, equations are analysed by drawing, which **display symmetries**

Noether's theorems:

conservation properties (energy, momentum) are
symmetries in the equations

They depend on the continuum (e.g. energy conservation is **continuous time translations**)

Computational tools

Object Oriented Programming allows :

- 1 - the **simulation** of discrete dynamics of **individual cells** in an organism or of organisms in a changing ecosystem:
no need of a pre-given phase space
- 2 - design very complex “**simulat**” as aggregation of algorithms
(Varenne)

Grounded on:

- **elaboration** of information (Turing, Church, Kleene ..)
- **transmission** of information (Shannon, Brillouin)

both information theories are grounded on fundamental invariants:

Independence from **hardware**, from **coding**

Revisting Symmetries

A **symmetry**: the set of **possible states** and the determination of a system **does not change** according to specific transformations (e.g. symmetries of a ball state on top of a mathematical hill)

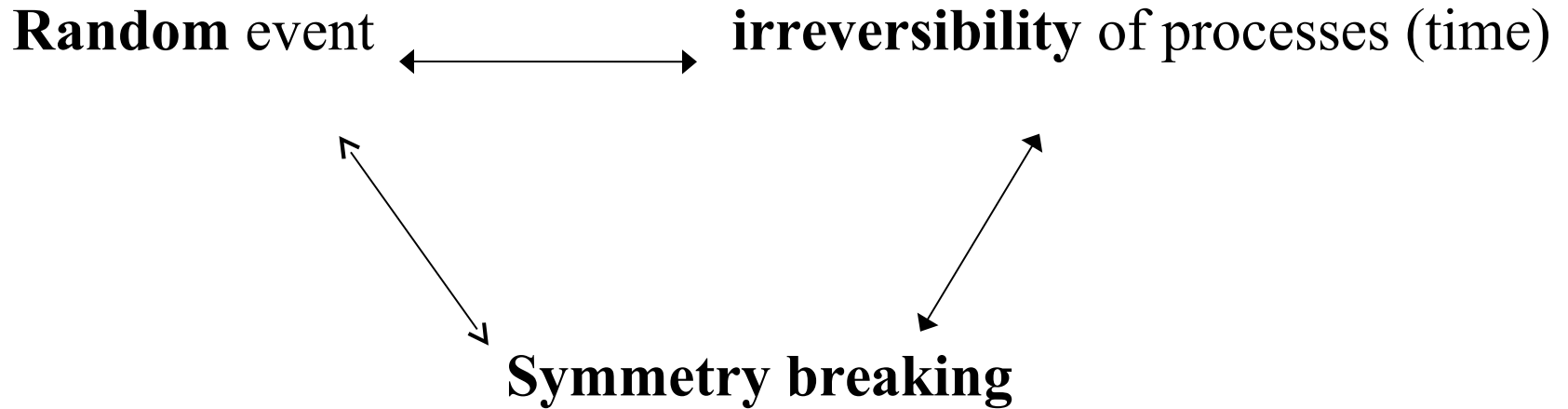
At the symmetry **breaking**: the state(s) becomes **no longer invariant** by these transformations (the trajectory goes to one of the formerly symmetric states and not to the others)

This is a physico-mathematical meaning to the philosophical notion of **contingency as non-necessity**.

Next: any random event (in all existing physical theorie) is
a symmetry breaking

Symmetries ?

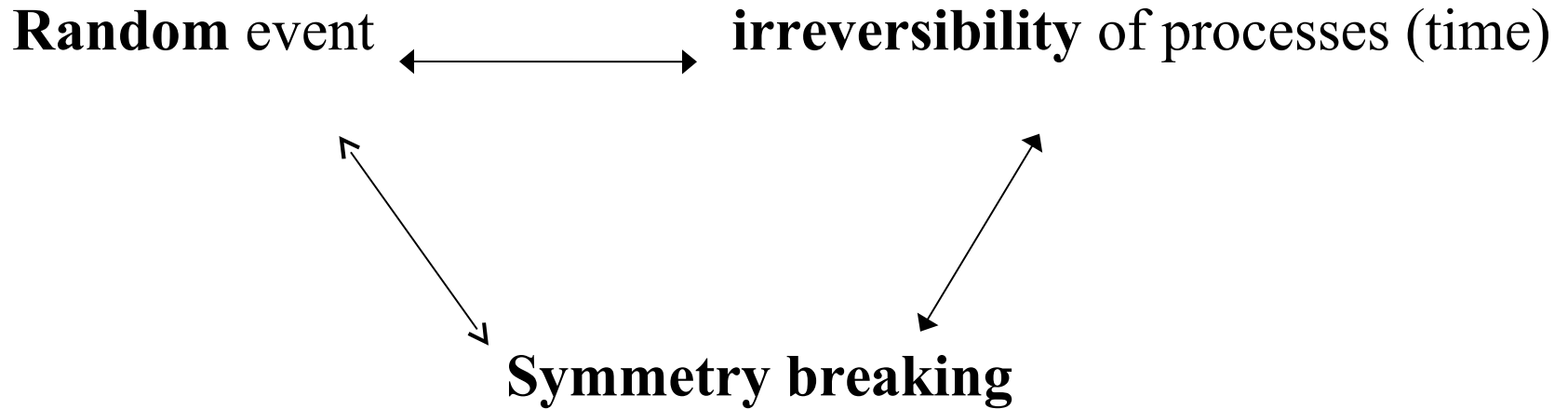
A triangular relation (in existing physical theories):



Longo G., Montévil M., **Perspectives on Organisms: Biological Time, Symmetries and Singularities**, *Springer*, 2013.

Symmetries ?

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A phylo(onto-)genetic trajectory is a **cascade of symmetry breakings**

Theoretical challenges in Biology

- *Mathematics* as a **science of invariants** and invariance preserving **transformations** (i.e. of “symmetries”): **Category Theory**
 - Continuous and discrete **symmetries**
 - **Equations** introduce symmetries and evolve with them

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 - a science of **iteration** (a new symmetry: *time* translation, from primitive *recursion* to *portability* of software):
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“**the perfect iteration of alpha-numeric re-writing rules**”
- *Biology*: a science of **structural stability** and **variability** (the main invariant?):
“**the never identical iteration of a morphogenetic process**”.

Agents in a Computational Field

From the Fribourg Coordination

***Coordination** is essential in all the activities where an ensemble of agents interacts within a distributed system.*

***Dynamics:** agents entering and exiting, adapt to new situations autonomous behavior.*

*Models for **coordination** - (i) direct coordination, (ii) connector-based coordination, (iii) shared data space, (iv) shared deductive knowledge base, (v) spatially distributed, locally interacting, propagating computational fields.*

*New interactions with **little or no need of communication protocols** for initialization.*

Agents in a Computational Field

From the Fribourg Coordination

*Co-fields are analogous to fields in physics: classical fields are scalars, vectors or tensors, which are functions of space and which are defined by partial differential equations with initial and/or boundary conditions. Analogously, **co-fields consist of suitable space dependent data structures** where **interaction** is possible only between neighbors.*

Question: what is a « biological field »?

Synchronous coordination and determination, but also

Historical formation...

Interactions that we better understand by continua

Spontaneous symmetry breaking

Some references (*downloadable*: Google: Giuseppe Longo)

- Bailly F., Longo G. **Mathematics and the Natural Sciences. The Physical Singularity of Life.** *Imperial College Press*, London, 2011 (français: Hermann, 2006).
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