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AGENT MODELS WITH INTERACTION

THE THERMODYNAMICS AND STATISTICAL MECHANICS PERSPECTIVES

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Interesting phenomena in social or economical sciences are traditionally considered difficult to study and almost impossible to predict at least in the sense of the hard sciences.

Among the reasons: sudden shifts in socio-economical behavior are observed in time or with respect to parameters.

It is clear that those abrupt swings difficult to predict are caused by peer to peer interaction rather than to external reasons.

Table 1. Prediction Success Table, Journey-to-Work

(Pre-BART Model and Post-BART Choices)

Cell Counts	Predicted Choices				
Actual Choices	**Auto Alone**	**Carpool**	**Bus**	**BART**	**Total**
Auto Alone	255.1	79.1	28.5	15.2	378
Carpool	74.7	37.7	15.7	8.9	137
Bus	12.8	16.5	42.9	4.7	77
BART	9.8	11.1	6.9	11.2	39
Total	352.4	144.5	94.0	40.0	631
Predicted Share	55.8%	22.9%	14.9%	6.3%	
(Std. Error)	*(11.4%)*	*(10.7%)*	*(3.7%)*	*(2.5%)*	
Actual Share	59.9%	21.7%	12.2%	6.2%	

(Source: McFadden 2001)

98%
agreement

Q: are there *predictive* theories in socio-economic sciences?

A: Discrete Choice Theory McFadden 1975-2000

Consider a "YES/NO" type question,

- civil or religious marriage

- in favor or against death penalty

Index individuals by i, $i = 1 \ldots N$

$$\sigma_i = \begin{cases} +1 \text{ if } i \text{ says YES} \\ -1 \text{ if } i \text{ says NO} \end{cases},$$

Minimize discomfort function

$$H_N(\sigma) = -\sum_{i=1}^{N} h_i \sigma_i.$$

$$h_i = \sum_{j=1}^{k} \alpha_j a_i^{(j)} + \alpha_0$$

where

$$a_i^{(1)} = \begin{cases} 1 \text{ for } i \text{ Male} \\ 0 \text{ for } i \text{ Female} \end{cases},$$

$$a_i^{(2)} = \begin{cases} 1 \text{ for } i \text{ Employee} \\ 0 \text{ for } i \text{ Self-employed} \end{cases},$$

If you add a noise (*logistic*) the search for minima is equivalent to the following probability distribution on the choices configurations:

$$P(\sigma) = \frac{e^{-\lambda H(\sigma)}}{\sum_{\sigma} e^{-\lambda H(\sigma)}}$$

Analogy with Boltzmann-Gibbs distribution in Statistical Mechanics.

Solution:

$$\bar{m}(a) = \mathrm{tanh}(\sum_{j=1}^{k} \alpha_j a^{(j)} + \alpha_0)$$

Works for public transportation, not for health insurance nor for many other cases.

Add interaction term! (Brock-Durlauf, 2000)

$$H_N(\sigma) \;=\; -\sum_{i,l=1}^{N} J_{il}\sigma_i\sigma_l - \sum_{i=1}^{N} h_i\sigma_i.$$

More complex system but still solvable. For instance in a model with 2 populations (Contucci-Ghirlanda)

$$\bar{m}_1 = \tanh(J_{11}\bar{m}_1 + J_{12}\bar{m}_2 + h_1)$$

$$\bar{m}_2 = \tanh(J_{21}\bar{m}_1 + J_{22}\bar{m}_2 + h_2)$$

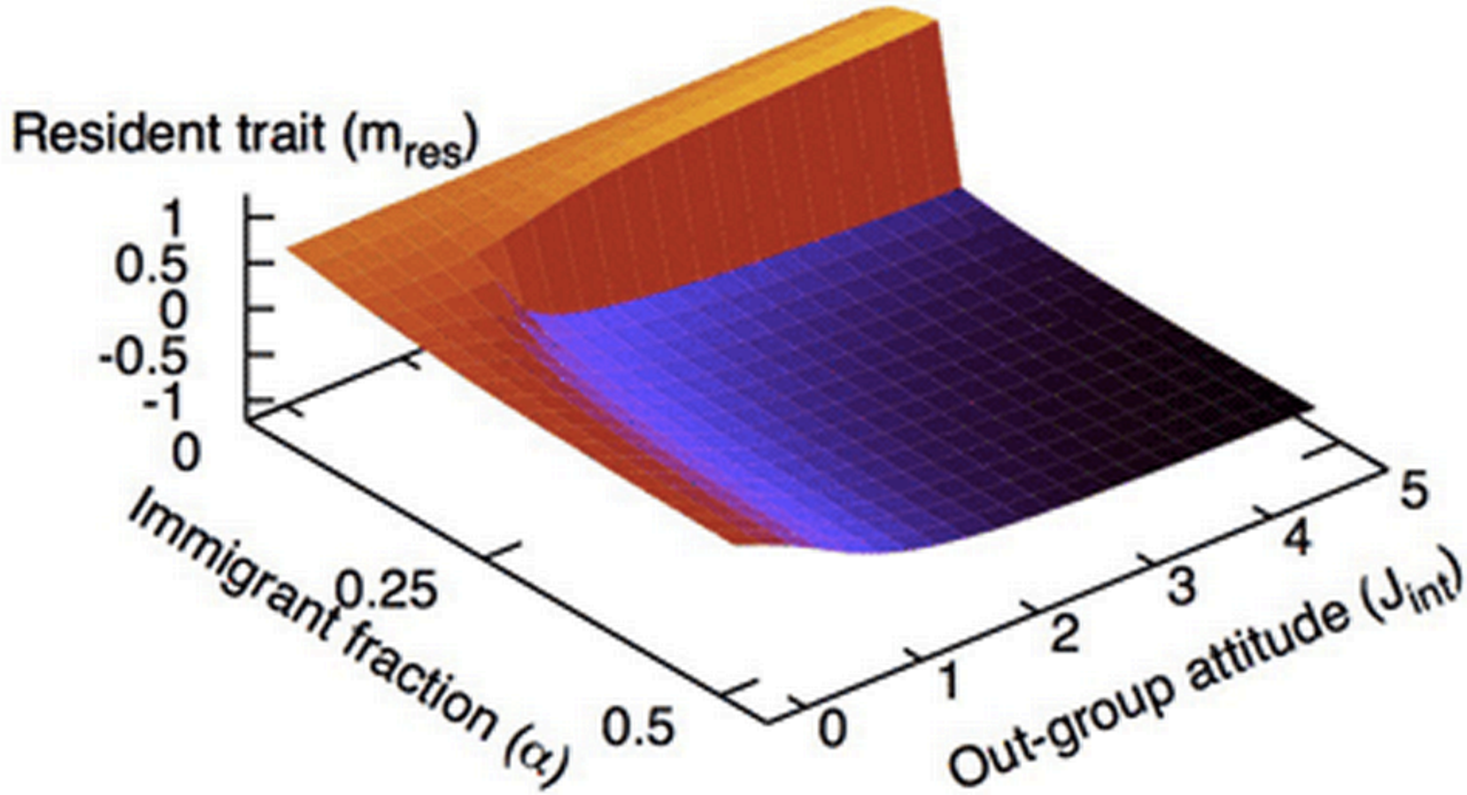

Figure 1: Influence of immigration on resident culture. The figure portrays the mean cultural trait after cultural contact as a function of the fraction of immigrants N1/N and the strength of the cross-group interaction J_int. Picture source: http://arxiv.org/abs/0712.1119

Q: Phenomenological evidence? Can we really distinguish imitation J from the individual inclinations h?

Max Weber (Economy and Society): *Social action is not identical with the similar actions of many persons... Thus, if at the beginning of a shower a number of people on the street put up their umbrellas at the same time, this would not ordinarily be a case of action mutually oriented to that of each other, but rather of all reacting in the same way to the like need of protection from the rain.*

The difficulty is to tune J and h.

Consider a two group system like immigrant-resident N_1, N_2. Observe coupling phenomena like labour market of permanent and temporary jobs, marriages and newborns. Study the integration quantifiers. Defining:

$$\gamma = \frac{N_1}{N_1 + N_2}$$

$$1 - \gamma = \frac{N_2}{N_1 + N_2}$$

the coupling volume is

$$N_1 N_2 \propto \gamma(1-\gamma)$$

Two theories emerge (joint wok by A.Barra, P.C., R.Sandell, C.Vernia),

free:

$$Q \ \propto \ \gamma(1-\gamma)$$

interacting:

$$Q \ \propto \ [\gamma(1-\gamma)]^{1/2}$$