Logic & Prolog

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Academic Year 2020/2021

1. Logic Programming
2. Prolog

Next in Line...

Origins I

Early history [Apt, 2005]

- automatic deduction of theorems
- *first-order logic* (FOL) by Frege, Peano and Russell
- computation as deduction by Gödel and Herbrand
- *resolution* principle by Robinson [Robinson, 1965], along with *unification*

The key issue

- resolution by Robinson
  - allowed proof of FOL theorem made it possible to compute with logic
  - not yet to see logic as a full computational framework
- from computable logic to logic as a programming language something was still missing

Origins II

The procedural interpretation of Horn clauses

- by defining logic programs as collections of Horn clauses
- by restricting Robinson's principle accordingly
- Kowalski showed how a logical implication could be amenable of both a *declarative* and a *procedural* interpretation [Kowalski, 1974]
- thus providing the *foundations* for a logic programming language
- Prolog, by Colmerauer in Marseille, came along in 1973

There is no question that Prolog is essentially a theorem prover à la Robinson. Our contribution was to transform that theorem prover into a programming language. [Colmerauer and Roussel, 1996]

Essentials I

Three fundamental features [Apt, 2005]

terms *Computing takes place over the domain of all terms defined over a "universal" alphabet.*

mgu *Values are assigned to variables by means of automatically-generated substitutions, called most general unifiers. These values may contain variables, called logical variables.*

backtracking *The control is provided by a single mechanism: automatic backtracking.*

Basic Units of Computation I

Atomic actions [Apt, 2005]

- logic programming is a different paradigm for programming languages
- since it is ruled by different *principles* w.r.t. the other sorts of programming languages
  - atomic actions are *equations* between *terms*
  - executed by means of the *unification* process trying to solve them
  - unification assigns *values* to *variables*
  - values can be *arbitrary terms*—in fact, there is just one sort of variable, ranging over the set of all terms
- so, in order to understand logic programming as a computational paradigm, we first need to understand its basic units of computation

Basic Units of Computation II

Terms: Definition

- a *variable* is a *term*
- a *functor* (or, *function symbol*) with arity 0 is called a *constant*, and is a term
- if f is a functor of arity n, and $t_1, \ldots, t_n$ are n terms, then $f(t_1, \ldots, t_n)$ is a term

Basic Units of Computation III

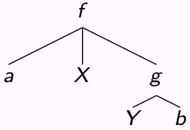
Terms: Examples

- let's say that X, Y are variables, a, b constants (or, functors of arity 0), f, g functors of arity $3, 2$ respectively. Then
  - a, b, X, and Y are proper terms
  - $f(a, b, a)$ and $g(X, Y)$ are proper terms
  - $f(a, X, g(Y, b))$ is a proper term
- variables and constant are *atomic terms*, terms built out of proper functors are *structured terms*. Then
  - a, b, X, and Y are atomic terms
  - $f(a, b, a)$, $g(X, Y)$ and $f(a, X, g(Y, b))$ are structured terms
  - ! in the structured term $f(a, X, g(Y, b))$, f is the *functor symbol* of *arity* 3, whereas $a, X, g(Y, b)$ are the three *subterms*

Basic Units of Computation IV

Terms: Remarks

- a *recursive* definition, leading to a recursive data structure—a tree
  - e.g., structured term $f(a, X, g(Y, b))$ maps onto tree



- fundamental in mathematical logic, terms are *essential in computer science*, too: e.g., they capture both arithmetic expressions and strings
- no specific alphabet is assumed—*universal alphabet* for all terms
- no meaning is a *a priori* attached to symbols, in particular to functors—e.g., $+$ is just a functor, not associated a priori with the plus sign of arithmetic

→ no types

Logic Programs I

Logic clause

- a logic clause is a (finite) disjunction of literals [Console et al., 1997]
- if $A_1, \ldots, A_n, B_1 \ldots, B_m$ are atoms, containing variables $X_1, \ldots, X_k$, then

$$\forall X_1, \ldots, X_k (A_1 \vee \ldots \vee A_n \vee \neg B_1 \vee \ldots \vee \neg B_m)$$

  is a logic clause, which is *logically equivalent* to

$$\forall X_1, \ldots, X_k ((A_1 \vee \ldots \vee A_n) \leftarrow (B_1 \wedge \ldots \wedge B_m))$$

  usually written simply as

$$A_1, \ldots, A_n \leftarrow B_1, \ldots, B_m$$

- a clausal normal form (CNF) is a *conjunction of clauses*

Logic Programs II

Definite clauses

- a definite clause, has just one positive literal ($n = 1$)

$$A \leftarrow B_1, \ldots, B_m$$

- a unitary clause, is a definite clause with no negative literal ($m = 0, n = 1$)

$$A \leftarrow$$

- a definite goal is a definite clause with no positive literal ($n = 0$)

$$\leftarrow B_1, \ldots, B_m$$

Horn clauses

- a Horn clause is either a definite clause or a definite goal ($n = 1$ or $n = 0$)

Logic Programs III

Logic program

- in a logic program
  - a definite clause is called a rule
  - a unitary clause is a fact
  - a definite goal is just a goal
- a logic program is a CNF of Horn clauses
  - so, it is a conjunction of rules and facts (and goals)

Goals & Proofs I

Proving goals

- Robinson's principle proceed by contradiction, trying to prove a formula F false against CNF H, succeeding if this fails
  - technically, proving that $H \cup \neg F$ is not satisfiable
- proving an atom G in logic programming amounts at proving $\neg G$ against logic program P
  - technically, proving goal $\leftarrow G$ on P
- computation in logic programming proceeds by *proving goals*

! resolution leads to *backward chaining*—from goal back to axioms

Goals & Proofs II

SLD resolution informally

- to prove a goal G w.r.t. program P, the resolution principle for logic programming proceeds according to the procedural interpretation
- so, first we look for one clause $A \leftarrow B_1, \ldots, B_n$ in P whose head A unifies with G
- if the most general unifier of G and A is θ ($mgu(G, A) = \theta$), then the proof of G succeeds if we can further prove $B_1\theta, \ldots, B_n\theta$—where $B_i\theta$ represents the application of the *mgu* θ to B_i

! the application of θ to clause $A \leftarrow B_1, \ldots, B_n$ specialises the clause to the specific atom we need to proof—that is, our current goal

! resolution proceed recursively with the proof of *subgoals* $B_1\theta, \ldots, B_n\theta$

→ in general, the computational state of the SLD resolution include a (possibly empty) conjunction of atom (goals) $G_1, \ldots, G_n$ to be proven—the *current goal* of the proof

An Example I

A simple logic program

$parent(joey, luca)$
$parent(joey, simone)$
$parent(lino, joey)$
$parent(mirella, joey)$

$grandparent(X, Z) \leftarrow parent(X, Y), parent(Y, Z)$

An Example II

Declarative interpretation

- four *facts* are expressed by means of predicate $parent/2$
  - four propositions that are considered true with no need of proof—our axioms
  - a possible interpretation is that, e.g., *joey* is a parent of *luca*—just one of the many, even though the most intuitive for English speakers
- one *rule* is expressed by means of predicate $grandparent/2$
  - since it is the short form for

    $$\forall X, Y, Z, grandparent(X, Z) \leftarrow parent(X, Y), parent(Y, Z)$$

    it means that formula $grandparent(X, Z)$ holds if both $parent(X, Y)$ and $parent(Y, Z)$ are true, whatever the values of X, Y, Z
  - so, it can be used to prove the truth of, e.g., formula $grandparent(lino, luca)$ since both $parent(joey, luca)$ and $parent(lino, joey)$ are true since they are facts in the logic program
  - independently of the possible interpretations

An Example III

Procedural interpretation

- two procedure are defined: *parent*/2 and *grandparent*/2
- two (procedure) calls can be executed correspondingly—goals of the form
  - $\leftarrow parent(?, ?)$
  - $\leftarrow grandparent(?, ?)$

  with any sort of term in the place of the ?
  - for instance, $\leftarrow grandparent(lino, luca)$
- to compute *parent*/2 we can use the four facts, non-deterministically
- to compute *grandparent*/2 we can use the rule, first matching the rule head, then proceeding by calling the two subprocedures, via the two *subgoals* of the form *parent*/2
  - for instance, to compute $\leftarrow grandparent(lino, luca)$ we will compute subgoals $\leftarrow parent(lino, Y)$ and $\leftarrow parent(Y, luca)$

An Example IV

Possible goals

- $grandparent(lino, luca)$ succeeds—one refutation, no *computed substitution*
- $grandparent(lino, joey)$ fails—no refutations
- $grandparent(lino, X)$ succeeds twice—two refutations, two different computed substitutions
  - $X/luca$
  - $X/simone$
- $grandparent(X, simone)$ succeeds twice—two refutations, two different computed substitutions
  - $X/lino$
  - $X/mirella$
- $grandparent(X, Y)$ succeeds four times—four refutations, four different computed substitutions
  - $X/lino$, $Y/luca$
  - $X/lino$, $Y/simone$
  - $X/mirella$, $Y/luca$
  - $X/mirella$, $Y/simone$

An Example: Remarks

Remarks

From the example we get some early hints about some benefits of logic programming

- multiple uses of the single program
  - the simple program above can be used to test the family relations between known people, or, to compute them
  - mostly, input / output parameters needs not to be defined a priori
- knowledge-based programming
  - arbitrarily complex relations expressed as FOL facts represent the core of a logic program
  - knowledge representation is straightforward in the logic programming formalism—with FOL
- language for rule-based systems
  - classical AI, such as expert systems [Buchanan and Shortliffe, 1984]

Next in Line. . .

1. Logic Programming

2. Prolog

Prolog examples: facts, atom & predicate I

Imagine we want to encode characters of Homer's Iliad and Odyssey.
→ This translates into Prolog *facts* ended with a period

```
character(priam, iliad).
character(hecuba, iliad).
character(achilles, iliad).
character(agamemnon, iliad).
character(patroclus, iliad).
character(hector, iliad).
character(andromache, iliad).
character(rhesus, iliad).
character(ulysses, iliad).
character(menelaus, iliad).
character(helen, iliad).
```

```
character(ulysses, odyssey).
character(penelope, odyssey).
character(telemachus, odyssey).
character(laertes, odyssey).
character(nestor, odyssey).
character(menelaus, odyssey).
character(helen, odyssey).
character(hermione, odyssey).
```

Prolog examples: facts, atom & predicate II

Fact

Facts are statements that describe object properties or relations between objects.

Knowledge

Such a collection of facts, and later, of rules, makes up a database. It transcribes the knowledge of a particular situation into a logical format. Adding more facts to the database, we express other properties

Prolog examples: facts, atom & predicate III

such as the gender of characters:

```
% Male characters
male(priam).
male(achilles).
male(agamemnon).
male(patroclus).
male(hector).
male(rhesus).
male(ulysses).
male(menelaus).
male(telemachus).
male(laertes).
male(nestor).
```

```
% Female characters
female(hecuba).
female(andromache).
female(helen).
female(penelope).
```

Prolog examples: facts, atom & predicate IV

or relationships between characters such as parentage:

```
% Fathers
father(priam, hector).
father(laertes, ulysses).
father(atreus, menelaus).
father(menelaus, hermione).
father(ulysses, telemachus).
```

```
% Mothers
mother(hecuba, hector).
mother(penelope, telemachus).
mother(helen, hermione).
```

Prolog examples: facts, atom & predicate V

Finally, if we want to describe kings of some cities and their parties, this would be done as:

```
king(ulysses, ithaca, achaean).
king(menelaus, sparta, achaean).
king(nestor, pylos, achaean).
king(agamemnon, argos, achaean).
king(priam, troy, trojan).
king(rhesus, thrace, trojan).
```

Prolog examples: facts, atom & predicate VI

General form of a Prolog fact

$$relation(object_1, object_2, ..., object_n).$$

Symbols or names representing objects → `ulysses` or `penelope`: *atoms*

Atoms

- strings of letters, digits, underscores begin with a lowercase letter
- can also be a string beginning with an uppercase letter or including white spaces, but it must be enclosed between quotes → `'Ulysses'` or `'Pallas Athena'` are legal atoms

Prolog examples: facts, atom & predicate VII

Predicate

- name of the symbolic relation is the *predicate*,
- the objects $object_1, object_2, ..., object_n$ involved in the relation are the *arguments*
- the number n of the arguments is the *arity*

Traditionally, a Prolog predicate is indicated by its name and arity

`predicate/arity`

→ e.g., `character/2`, `king/3`

Prolog examples: terms I

Terms

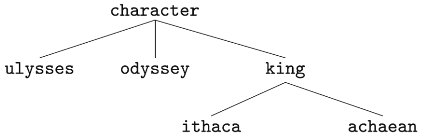
- all forms of data are called *terms*
- constants, i.e., atoms or numbers are terms
- facts, like `king(menelaus, sparta, achaean)`, are a *compound term* or a structure, that is, a term composed of other terms (called subterms)

→ arguments of this compound term are constants

→ can also be other compound terms, as in `character(priam, iliad, king(troy, trojan))` where the arguments of the predicate character/3 are two atoms and a compound term

Prolog examples: tree of terms

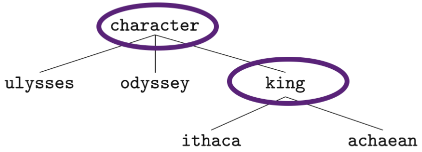
```
character(ulysses, odyssey, king(ithaca, achaean))
```



- nodes of the tree are equivalent to the functors of a term

Prolog examples: tree of terms

```
character(ulysses, odyssey, king(ithaca, achaean))
```



- use trees to represent compound terms
- nodes of the tree are equivalent to the functors of a term

Prolog examples: compound terms

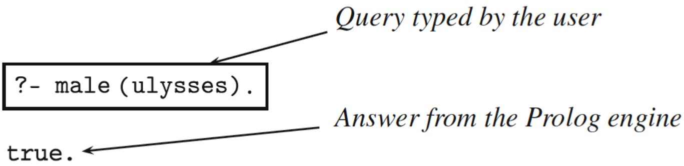
Compound Term

- functor – the name of the relation – and arguments
- leftmost functor of a term is the principal functor
- same principal functor with a different arity corresponds to different predicates: `character/3` is thus different from `character/2`
- constant is a special case of compound term with no arguments and an arity of 0 (can be referred to as `abc/0`)

Prolog examples: query I

Query

- request to prove or retrieve information from the knowledge base
- Prolog answers yes if it can prove it, that is, here if the fact is in the database, or no if it cannot: if the fact is absent (case of asking for a fact tu be proven)
- question *Is Ulysses a male?*



Prolog examples: query II

```
?- male(penelope).
false.
```

- expressions `male(ulysses)` or `male(penelope)` are *goals* to prove
- some questions require more goals, such as *Is Menelaus a male and is he the king of Sparta and an Achaean?*

```
?- male(menelaus), king(menelaus, sparta, achaean).
true.
```

- where "," is the conjunction operator → indicates that Prolog has to prove both goals
- compound queries: conjunction of two or more goals
  `?- G1, G2, G3, ..., Gn.`
- Prolog proves that all the goals $G_1 ... G_n$ are true

Prolog examples: logical variables I

Logical variables

- begin with an uppercase letter, for example, `X`, `Xyz`, or an underscore
- stand for any term: constants, compound terms, and other variables
- term containing variables such as `character(X, Y)` can *unify* with a compatible fact, such as `character(penelope, odyssey)`, with the *substitutions* `X = penelope` and `Y = odyssey`
- Prolog resolution algorithm searches terms in the database that unify with it → substitutes the variables to the matching arguments
- question *What are the characters of the Odyssey?*

Prolog examples: logical variables II

The variable *The query*

```
?- character(X, odyssey).
```

The Prolog answer

```
X = ulysses
```

Prolog examples: logical variables III

- question *What is the city and the party of king Menelaus?*

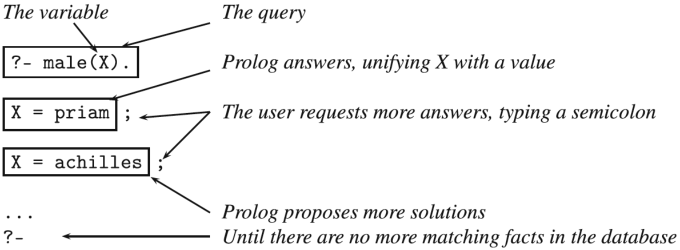
```
?- king(menelaus, X, Y).
X = sparta, Y = achaean

?- character(menelaus, X, king(Y, Z)).
X = iliad, Y = sparta, Z = achaean

?- character(menelaus, X, Y).
X = iliad, Y = king(sparta, achaean)
```

- multiple solutions → Prolog considers the first fact to match the query in the knowledg
- type ";" to get the next answers until there is no more solution

Prolog examples: logical variables IV



Prolog examples: shared variables I

Shared variables

- goals in a conjunctive query can share variables
- constrain arguments of different goals to have the same value
- question *Is the king of Ithaca also a father?*
- conjunction of two goals `king(X, ithaca, Y)` and `father(X, Z)`, with `X` shared between the goals

```
?- king(X, ithaca, Y), father(X, Z).
X = ulysses, Y = achaean, Z = telemachus
```

Prolog examples: shared variables II

- not interested in the name of the child although Prolog responds with `Z = telemachus`.
- can indicate we do not need to know the values of Y and Z using anonymous variables

```
?- king(X, ithaca, _), father(X, _).
X = ulysses
```

Prolog examples: rules I

Rules

- enable to derive a new property or relation from a set of existing ones
- a term called the *head* or *consequent* followed by symbol `:-` (read if) and a conjunction of goals called *antecedent* or *body*

$$HEAD :- G_1, G_2, G_3, \ldots G_n.$$

- the head is true if the body is true
- variables are shared between the body and the head

Prolog examples: rules II

```
son(X, Y) :- father(Y, X), male(X).
son(X, Y) :- mother(Y, X), male(X).

?- son(telemachus, Y).
Y = ulysses;
Y = penelope;
```

Rules

Flexible way to *deduce* new information from a set of facts.

The `parent/2` predicate is another example of a family relationship that is easy to define using rules. Somebody is a parent if s/he is either a mother or a father:

```
parent(X, Y) :- mother(X, Y).
parent(X, Y) :- father(X, Y).
```

Prolog examples: rules III

- Rules can call other rules as with `grandparent/2`
- Z is an intermediate variable shared between goals. $\rightarrow$ enables to find the link between the grandparent and the grandchild: a mother or a father

```
grandparent(X, Y) :- parent(X, Z), parent(Z, Y).
```

- generalize the `grandparent/2` predicate and write `ancestor/2`
- two rules, one of them being recursive

```
ancestor(X, Y) :- parent(X, Y).
ancestor(X, Y) :- parent(X, Z), ancestor(Z, Y).
```

Prolog examples: rules IV

- recursion pattern is quite common for Prolog rules
- one or more rules express a general case using recursion
- another set of rules or facts describes simpler conditions without recursion $\rightarrow$ correspond to boundary cases and enable the recursion to terminate

Prolog clauses

Facts and rules are also called *clauses*

Prolog Program

Prolog program

program	a sequence of Prolog clauses
	interpreted as a *conjunction* of clauses
logic theory	constituting a *logic theory* made of Horn clauses written according the Prolog syntax

Prolog Execution I

Aim of a Prolog computation

- given a Prolog program P and the goal `?- p(t1,t2,...,tm)` (also called *query*)
- if `X1,X2,...,Xn` are the variables in terms `t1,t2,...,tm`
- the meaning of the goal is to query P and find whether there are some values for `X1,X2,...,Xn` that make `p(t1,t2,...,tm)` true

→ thus, the aim of the Prolog computation is to find a substitution $\sigma =$`X1/s1,X2/s2,...,Xn/sn` such that $P \models \mathtt{p}(\mathtt{t1}, \mathtt{t2}, \ldots, \mathtt{tm})\sigma$

Prolog Execution II

Prolog search strategy

- as a logic programming language, Prolog adopts SLD resolution
- as a search strategy, Prolog applies resolution in a strictly linear fashion
  - *goals* are replaced *left-to-right*, sequentially
  - *clauses* are considered in *top-to-bottom* order
  - *subgoals* are considered *immediately* once set up

→ resulting in a *depth-first* search strategy

Prolog Execution III

Prolog backtracking

- in order to achieve completeness, Prolog saves choicepoints for any possible alternative still to be explored
- and goes back to the nearest choice point available in case of failure
- exploiting automatic backtracking

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